1. Evaluate the following limits. [Show all work - no shortcuts].

(a). $\displaystyle \lim_{x\to\infty} \frac{3 - x^2 + 4x^3}{x^4 + 2x}$ **(b).** $\displaystyle \lim_{x\to-\infty} \frac{2x^2 + 3x + 1}{3x^2 - 3x - 4}$

2. Find the horizontal asymptote(s) of the following function. [You may use shortcuts.] $f(x) = \dfrac{2x+1}{\sqrt{4x^2 - x}}$.

3. Given the following function and its derivatives

$$f(x) = \frac{x}{9 - x^2} \qquad f'(x) = \frac{x^2 + 9}{(9 - x^2)^2} \qquad f''(x) = \frac{2x(x^2 + 27)}{(9 - x^2)^3}$$

Use the Summary of Curve Sketching to determine the relevant information. Sketch the graph of the function. Label any maximum and minimum points and inflection points.

domain:

x-intercept(s):

y-intercept:

vertical asymptote(s):

horizontal asymptote(s):

slant asymptote:

critical numbers:

intervals where increasing:

intervals where decreasing:

coordinates of local max/min(s):

intervals where concave up:

intervals where concave down:

inflection point(s):

4. If a box with a square base and open top is to hold 4 ft^3, find the dimensions of the box that will require the least amount of material.

5. Find the maximum possible volume of a right circular cylinder if its total surface area (including top and bottom) is 150π.

6. A Norman window is constructed by adjoining a semicircle to the top of an ordinary rectangular window. Find the dimensions of the Norman window with the largest possible area if the total perimeter is 16 ft.

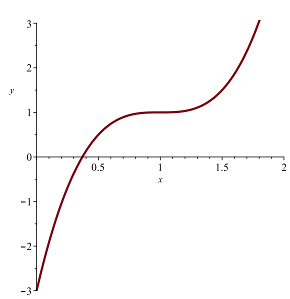
7. Given $f(x) = 4x^3 - 12x^2 + 12x - 3$, [The graph is given below.]

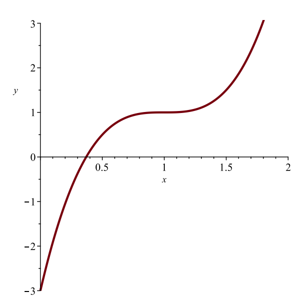
(a). Explicitly write out Newton's formula for finding the root of this function.

(b). Start with an initial guess of $x_0 = 0.5$ and iterate Newton's Method once. [***Do <u>not</u> simplify your answer!***]

(c). Starting with $x_0 = 0.5$, demonstrate Newton's method by marking $x_0, x_1, x_2, \dots$ and the associated tangent lines on the graph of $f(x)$. Does it seem like Newton's method will work if you start with this initial guess?

(d). Starting with $x_0 = 1.0$, demonstrate Newton's method by marking $x_0, x_1, x_2, \dots$ and the associated tangent lines on the graph of $f(x)$. Does it seem like Newton's method will work if you start with this initial guess? Why or why not?





8. Find the antiderivatives for the following functions.

(a). $h(x) = 3x^3 - 7x^2$

(b). $f(x) = \sqrt{x} - \sqrt{3}x^2$

9. Given that $g'(\theta) = -\sec^2\theta$ and $g\left(\frac{\pi}{3}\right) = 0$, find $g(\theta)$.

10. Given the function $f(x) = \frac{3}{x}$, estimate the area under the curve $f(x)$ on the interval $[1, 6]$ using 5 subintervals and using the right endpoint of each subinterval. [i.e. find R_5].

11. Using the definition of the definite integral $\displaystyle\int_a^b f(x)\,dx = \lim_{n\to\infty}\sum_{i=1}^{n} f(x_i)\Delta x = \lim_{n\to\infty} R_n$, ***set-up, but do not evaluate***, the summation/limit using right endpoints for the integral $\displaystyle\int_0^1 x^3 + 1\,dx$.

12. Section 4.3: #3

13. Evaluate the following integrals [Use integration techniques, ***not*** the limit definition.]:

(a). $\displaystyle\int_1^2 t + 2\,dt$

(b). $\displaystyle\int_1^{x^2} t + 2\,dt$

(c). $\displaystyle\int_0^4 \frac{x(2+x)}{\sqrt{x}}\,dx$

14. Use the Fundamental Theorem of Calculus (Part B/1) to find $F'(x)$

(a). $F(x) = \displaystyle\int_0^x t\cos t\,dt$

(b). $F(x) = \displaystyle\int_{-2}^{x^2} \sqrt{t+8}\,dt$

15. A particle moves with a velocity of $v(t) = -t^2 + 4t$ on the interval $0 \le t \le 6$.

(a). Find the displacement

(b). Find the total distance traveled

16. Let $r(t)$ be the rate at which the world's oil is consumed, where t is measured in years starting at $t = 0$ on January 1, 2000 and $r(t)$ is measured in barrels per year. What does $\int_0^8 r(t)\,dt$ represent and what are its units?

17. Evaluate the following integrals. [**Note:** You may or may not need to use substitution.] Check your answer by differentiating the result.

(a). $\displaystyle\int 3x^5 - 4x^3 + 6x + 2\,dx$

(b). $\displaystyle\int (3x-1)(3x^2-2x)^2\,dx$

(c). $\displaystyle\int x(3x^2-2x)^2\,dx$

(d). $\displaystyle\int \left(1 + \frac{1}{t}\right)\left(\frac{1}{t^2}\right)\,dt$

(e). $\displaystyle\int_0^{\pi/6} \sec x\tan x\,dx$

(f). $\displaystyle\int \sin x\cos x\,dx$

(g). $\displaystyle\int \frac{5x}{\sqrt[3]{1-x^2}}\,dx$

(h). $\displaystyle\int_1^3 \frac{x^2+1}{x^2}\,dx$

(i). $\displaystyle\int (2-x)^6\,dx$

(j). $\displaystyle\int \theta\sin(3\theta^2)\,d\theta$