Name: Key

Math 151, Calculus I – Crawford

Exam 3

19 May 2019

	Score
1	/8
2	/14
3	/12
4	/8
5	/8
6	/10
7	/28
8	/12
9	/2
Total	/100

- Calculators, books, notes (in any form), cell phones, and any unauthorized sources are ***not*** allowed.
- You may use the given Unit Circle.
- Clearly indicate your answers.
- ***Show all your work*** – partial credit may be given for written work.
- ***Good luck***!

1. (8 pts). Find the horizontal asymptote(s), if any, of the following function. [Justify your answer.]

$$f(x) = \frac{1-2x+4x^2}{3x^2-1}$$

$$\lim_{x\to\infty} \frac{1-2x+4x^2}{3x^2-1} = \frac{4}{3}$$

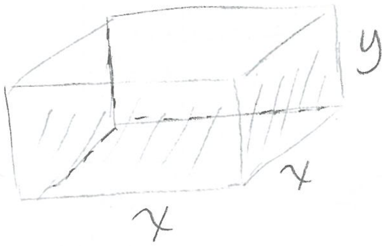
So $\boxed{y = \frac{4}{3} \text{ is H.A.}}$

$$\lim_{x\to-\infty} \frac{1-2x+4x^2}{3x^2-1} = \frac{4}{3}$$

2. (14 pts). Suppose there are 27 ft^2 of material available to make a box with square base and an open top. Find the dimensions of such a box that will have the largest possible volume.

[Remember that significant partial credit will be given for clearly and accurately labeling a picture and setting up the problem.]

$V = x^2 y \qquad A = \underbrace{4xy + x^2 = 27}$



Maximize $\underline{V = x^2 y}$ subject to $\underline{4xy + x^2 = 27}$

$4xy = 27 - x^2$

$y = \dfrac{27 - x^2}{4x}$

Area of each side xy

Area of bottom x^2

(No top)

$V = x^2\left(\dfrac{27 - x^2}{4\cancel{x}}\right)$

$= \dfrac{x}{4}(27 - x^2)$

$V = \dfrac{27}{4}x - \dfrac{1}{4}x^3$

$V' = \dfrac{27}{4} - \dfrac{3}{4}x^2 = 0$

$\dfrac{3}{4}x^2 = \dfrac{27}{4}$

$x^2 = \dfrac{\cancel{27}^{9}}{\cancel{4}} \cdot \dfrac{\cancel{4}}{\cancel{3}}$

$x^2 = 9$

$x = \pm 3$

$x = 3$

$y = \dfrac{27 - (3)^2}{4(3)}$

$= \dfrac{27 - 9}{12}$

$= \dfrac{18}{12}$

$= \dfrac{3}{2}$

$\boxed{x = 3\text{ft} \quad y = \tfrac{3}{2}\text{ft} \quad \text{or} \quad 3' \times 3' \times \tfrac{3}{2}'}$

3. (12 pts). Given that $f''(x) = 2 - 3x$ and $f'(0)=2, f(1)=0$ find $f(x)$.

$f'(x) = 2x - \frac{3}{2}x^2 + C$

$f'(0) = 2(0) - \frac{3}{2}(0)^2 + C = 2$

$C = 2$

$f'(x) = 2x - \frac{3}{2}x^2 + 2$

$\Rightarrow f(x) = x^2 - \frac{1}{2}x^3 + 2x + D$

$f(1) = 1^2 - \frac{1}{2}(1)^3 + 2(1) + D = 0$

$1 - \frac{1}{2} + 2 + D = 0$

$\frac{2-1+4}{2} + D = 0$

$\frac{5}{2} + D = 0$

$D = -\frac{5}{2}$

$$\boxed{f(x) = x^2 - \frac{1}{2}x^3 + 2x - \frac{5}{2}}$$

4. (8 pts). Determine a definite integral $\int_a^b f(x)\,dx$ that is equivalent to the given limit.

[Recall $\int_a^b f(x)\,dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x$]

$$\lim_{n\to\infty} \sum_{i=1}^{n} \Bigg[\overbrace{\left(1+\frac{3}{n}i\right)^2 - \left(1+\frac{3}{n}i\right)}^{x_i^2 - x_i}\Bigg]\frac{3}{n}$$

$\leftarrow f(x) = x^2 - x$

$\Delta x = \frac{3}{n} = \frac{b-a}{n}$

So $b - a = 3$

$x_i = a + i\Delta x$

$= a + i \cdot \frac{3}{n}$

$= 1 + \frac{3}{n}i$

So $a = 1$

$a = 1$ & $b - a = 3$

$\Rightarrow b - 1 = 3$

$b = 4$

$$\boxed{\int_1^4 x^2 - x \, dx}$$

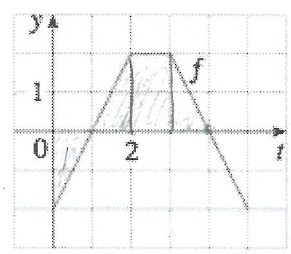
5. (8 pts). Use the Fundamental Theorem of Calculus Part B/1 to find $F'(x)$ for

$$F(x) = \int_{-2}^{\sqrt{x}} \sin t \, dt$$

$F'(x) = \sin\sqrt{x} \cdot \frac{d}{dx}[x^{1/2}]$

$= (\sin\sqrt{x}) \cdot \frac{1}{2} x^{-1/2} = \boxed{\frac{1}{2\sqrt{x}} \sin\sqrt{x}}$

6. (10 pts). Let $g(x) = \int_0^x f(t)\, dt$ where f is the function graphed below.



(a). Find $g(0)$ and $g(4)$.

$g(0) = \int_0^0 f(t)dt = \boxed{0}$

$g(4) = \int_0^4 f(t)dt$

$= -A_1 + A_2 + A_3 + A_4$

$= -\frac{1}{2}(1)(2) + \frac{1}{2}(1)(2) + (1)(2) + \frac{1}{2}(1)(2)$

$= 2 + 1 = \boxed{3}$

(b). On what interval(s) is g increasing?

g increases when g' is positive

$g'(x) = \frac{d}{dx}\left[\int_0^x f(t)dt\right]$

$= f(x)$

So g' is positive when f is positive ⇒ $\boxed{\text{Interval } (1,4)}$

7. (28 pts). Evaluate the following integrals. [Use integration techniques, ***NOT*** the limit definition.]

(a). $\int x^2\sqrt{x} - 5x + 6\,dx$ $= \int x^2 \cdot x^{1/2} - 5x + 6\,dx$

$= \int x^{5/2} - 5x + 6\,dx$

$= \boxed{\frac{2}{7}x^{7/2} - \frac{5}{2}x^2 + 6x + C}$

(b). $\int_{\pi/6}^{\pi/3} \sin x\,dx$ $= -\cos x \Big|_{\pi/6}^{\pi/3}$

$= -\cos(\pi/3) + \cos(\frac{\pi}{6})$

$= \boxed{-\frac{1}{2} + \frac{\sqrt{3}}{2} = \frac{\sqrt{3}-1}{2}}$

(c). $\int \frac{x^2+1}{(x^3+3x)^4}\,dx$

$u = x^3 + 3x$

$du = (3x^2+3)\,dx$

$\frac{1}{3}du = (x^2+1)\,dx$

$= \frac{1}{3}\int \frac{1}{u^4}\,du$

$= \frac{1}{3}\int u^{-4}\,du = \frac{1}{3}\frac{u^{-3}}{-3} + C = -\frac{1}{9}\cdot\frac{1}{u^3} + C$

$= \boxed{-\frac{1}{9}\cdot\frac{1}{(x^3+3)^3} + C}$

8. (12 pts). The velocity of a particle is given below. Find the <u>total distance traveled</u> over $0 \le t \le 2$.

$v(t) = t^2 - 1.$

$v(t) = t^2 - 1 = 0$

$t^2 = 1$

$t = \pm 1$

only $t = 1$ in interval

$\int_0^1 t^2 - 1\,dt = \frac{1}{3}t^3 - t \Big|_0^1 = \frac{1}{3} - 1 - (0) = -\frac{2}{3}$ (neg direction)

$\int_1^2 t^2 - 1\,dt = \frac{1}{3}t^3 - t \Big|_1^2 = \frac{1}{3}(2)^3 - 2 - \left(\frac{1}{3} - 1\right)$

$= \frac{8}{3} - 2 - \left(-\frac{2}{3}\right)$

$= \frac{8}{3} - 2 + \frac{2}{3} = \frac{10}{3} - 2 = \frac{10-6}{3} = \frac{4}{3}$

(pos. direction

Total Distance:

$\left|-\frac{2}{3}\right| + \left|\frac{4}{3}\right| = \frac{2}{3} + \frac{4}{3} = \frac{6}{3} = \boxed{2}$

9. (2 pts). $\boxed{\text{TRUE}}$ or FALSE: If Newton's method is used to find the root of the equation $x - \cos(3x) = 0$, the equation for Newton's Method is given by

$$x_{n+1} = x_n - \frac{x_n - cos(3x_n)}{1 + \sin(3x_n) \cdot 3}$$

$f(x) = x - \cos(3x)$

$f'(x) = 1 + \sin(3x) \cdot 3$

$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$