1. Differentiate the following using ***Differentiation Rules***.

(a). $s(t) = \sqrt[3]{(t^3 + 3t^2 + 4)^2}$ $\quad s'(t) = \frac{2}{3}(t^3 + 3t^2 + 4)^{-1/3} \cdot (3t^2 + 6t) = \frac{2(3t^2 + 6t)}{3\sqrt[3]{t^3 + 3t^2 + 4}}$

(b). $s(t) = (3t^3 - t^2 + 7)^{23}$ $\quad s'(t) = 23(3t^3 - t^2 + 7)^{22}(9t^2 - 2t)$

(c). $f(\theta) = \theta \sin(\theta^2 + 1)$ $\quad f'(\theta) = 2\theta^2 \cos(\theta^2 + 1) + \sin(\theta^2 + 1)$

(d). $y = \dfrac{x(2x^4 + 4)^8}{\tan 2x}$ [Do not simplify!] $\quad \frac{dy}{dx} = \frac{\tan 2x \cdot \left[x \cdot 8(2x^4 + 4)^7 \cdot 8x^3 + (2x^4 + 4)^8 \cdot 1\right] - x(2x^4 + 4)^8(\sec^2(2x) \cdot 2)}{\tan^2 2x}$

2. Find the equation of the tangent line to the curve $y = \sqrt[3]{2x^2 - 5}$ at $x = 4$. $\quad y - 3 = \frac{16}{27}(x - 4)$

3. Given $f(x) = g(3x^2)$, find f' in terms of g'. $\quad f'(x) = g'(3x^2) \cdot 6x$

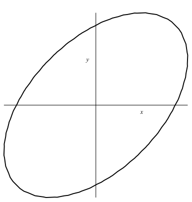
4. Given the curve drawn below and defined by $\quad x^2 + y^2 = 3 + xy$

(a). Find $\dfrac{dy}{dx}$ $\quad \frac{dy}{dx} = \frac{y - 2x}{2y - x}$

(b). On the graph below, sketch any tangents lines to the curve where the slope is 0.

(c). Use part **(a)** to find these points on the curve where the slope is 0. Must show work for credit. $\quad (1, 2)$ & $(-1, -2)$.

(d). Find $\dfrac{d^2y}{dx^2}$ in terms of x and y. $\quad \frac{d^2y}{dx^2} = \frac{(2y - x)\left(\frac{y - 2x}{2y - x} - 2\right) - (y - 2x)\left(2\frac{y - 2x}{2y - x} - 1\right)}{(2y - x)^2}$



5. A tank holds 1000 gallons of water, which drains from the bottom of the tank in 50 minutes. Torricelli's Law gives the volume V of water remaining in the tank after t minutes as $V = 1000\left(1 - \frac{1}{50}t\right)^2$ for $0 \le t \le 50$. Find the rate at which the water is draining from the tank after 10 minutes. Include units in your answer. $\quad$ -32 gallons/min.

6. The cost function for a certain commodity is $C(x) = 60 + 0.12x - 0.0004x^2 + .000002x^3$. [Calculator*]

(a). Find the marginal cost function. $\quad C'(x) = 0.12 - 0.0008\,x + 0.000006\,x^2$

(b). Find and interpret $C'(50)$. $\quad C'(50) = \$0.095$.

The rate the cost is changing when the 50th item is produced is approximately $0.095 per item.

(c). Compare $C'(50)$ with the cost of producing the 51st item. $\quad C(51) - C(50) = \$0.094902$

7. If a stone is thrown vertically upward on the moon with a velocity of 8 m/s, its height after t seconds is given by $y = 8t - 0.83t^2$, [Calculator*]

(a). What is the velocity after 2 s? 4.68 m/s

(b). What is the velocity at impact? -8 m/s

8. Any Section 2.7 applications.

9. Given $f(x) = \sqrt{x} = x^{1/2}$

(a). Find the linearization $L(x)$ at $a = 25$ $L(x) = 5 + \frac{1}{10}(x - 25)$

(b). Use this linearization $L(x)$ to approximate $\sqrt{24.7}$ [Simplify your answer.] 4.97

10. A ladder 8 feet long is leaning against the wall of a house. On the ground, the base of the ladder is being pulled away from the wall at a rate of $\frac{3}{2}$ ft/sec. How fast is the angle between the top of the ladder and the wall changing when the this angle is $\frac{\pi}{3}$. $\frac{3}{8}$ rad/s

11. A particle moves along the curve $xy^2 = 12$. As it reaches the point $(3, 2)$, the y-coordinate is decreasing at a rate of 2 cm/s. How fast is the x-coordinate of the particle position changing changing at that instant? 6 cm/s

12. Find the critical numbers for $g(t) = 4t^3 - 3t^2 + 1$ $t = 0, \frac{1}{2}$

13. Given $f(x) = 2\sqrt{x} - x$, find the absolute maximum and minimum *values* of $f(x)$ on the interval $[0, 9]$.

Absolute Max = 1 at $x = 1$; Absolute Min = -3 at $x = 9$.

14. Given $f(x) = \frac{(x-1)^3}{x^2}$ $f'(x) = \frac{(x-1)^2(x+2)}{x^3}$ $f''(x) = \frac{6(x-1)}{x^4}$

(a). Find the intervals of increase or decrease. Increasing: $(-\infty, -2) \cup (0, 1) \cup (1, \infty)$ Decreasing: $(-2, 0)$

(b). Find the local maximum and minimum values. No Min ; Max $= -\frac{27}{4}$ at $x = -2$.

(c). Find the intervals of concavity and the inflection points. Up: $(1, \infty)$ Down: $(-\infty, 0) \cup (0, 1)$ Inf. Pts.: (1,0)

15. Given $f(\theta) = \cos^2(\theta)$, on $0 \le \theta \le 2\pi$,

(a). Find the intervals of increase or decrease. Increasing: $\left(\frac{\pi}{2}, \pi\right) \bigcup \left(\frac{3\pi}{2}, 2\pi\right)$ Decreasing: $\left(0, \frac{\pi}{2}\right) \bigcup \left(\pi, \frac{3\pi}{2}\right)$

(b). Find the local maximum and minimum values. Min = 0 at $x = \frac{\pi}{2}, \frac{3\pi}{2}$ and Max = 1 at $x = \pi$.

(c). Find the intervals of concavity and the inflection points.

Up: $\left(\frac{\pi}{4}, \frac{3\pi}{4}\right) \bigcup \left(\frac{5\pi}{4}, \frac{7\pi}{4}\right)$ Down: $\left(0, \frac{\pi}{4}\right) \bigcup \left(\frac{3\pi}{4}, \frac{5\pi}{4}\right) \bigcup \left(\frac{7\pi}{4}, 2\pi\right)$ Inf. Pts.: $\left(\frac{\pi}{4}, \frac{1}{2}\right), \left(\frac{3\pi}{4}, \frac{1}{2}\right), \left(\frac{5\pi}{4}, \frac{1}{2}\right), \left(\frac{7\pi}{4}, \frac{1}{2}\right)$

16. Section 3.3 #27

*Similar non-calculator problems could be given.