Name: Key

Math 151, Calculus I – Crawford

Exam 2

22 October 2019

Score

1	/12
2	/16
3	/12
4	/12
5	/10
6	/14
7	/12
8	/14
Total	/100

- Calculators, books, notes (in any form), cell phones, and any unauthorized sources are ***not*** allowed.
- You may use the attached unit circle. Simplify trigonometric functions at all standard values.
- Clearly indicate your answers.
- ***Show all your work*** – partial credit may be given for written work.
- ***Good luck***!

1. (12 pts). Find an equation of the tangent line to $f(x) = x(x^2+1)^3$ at $x = 1$.

① pt: $f(1) = 1(1^2+1)^3 = 1(2)^3 = 8 \Rightarrow (1,8)$

② Slope: $f'(x) = x \cdot 3(x^2+1)^2 \cdot 2x + (x^2+1)^3 \cdot 1$

$= 6x^2(x^2+1)^2 + (x^2+1)^3$

$f'(1) = 6(2)^2 + (2)^3 = 24 + 8 = 32 = m$

$$\boxed{y - 8 = 32(x-1)}$$

2. (16 pts). Differentiate the following **[Do not simplify!]**

(a). $g(\theta) = \sin^4(2\theta)$ $= (\sin(2\theta))^4$

$g'(\theta) = 4(\sin(2\theta))^3 \cdot \frac{d}{d\theta}[\sin(2\theta)]$ CR

$= \boxed{4\sin^3(2\theta)\cdot\cos(2\theta)\cdot 2}$

$= 8\cos(2\theta)\sin^3(2\theta)$

(b). $y = \sqrt{\dfrac{5x-1}{x^2+4}}$ $= \left(\frac{5x-1}{x^2+4}\right)^{1/2}$

$$\boxed{y' = \frac{1}{2}\left(\frac{5x-1}{x^2+4}\right)^{-1/2} \cdot \frac{(x^2+4)(5)-(5x-1)(2x)}{(x^2+4)^2}}$$

3. (12 pts). Use implicit differentiation to find y' for the given curve.

$x^2 + \tan y = y + xy^3$

$\frac{d}{dx}[x^2 + \tan y] = \frac{d}{dx}[y + xy^3]$

$2x + \sec^2 y \cdot y' = y' + x\cdot 3y^2 y' + y^3 \cdot 1$

$\sec^2 y \cdot y' - y' - 3xy^2 \cdot y' = y^3 - 2x$

$y'(\sec^2 y - 1 - 3xy^2) = y^3 - 2x$

$$\boxed{y' = \frac{y^3 - 2x}{\sec^2 y - 1 - 3xy^2}}$$

4. (~~12~~ 14 pts). Given $f(x) = \sqrt{x}$ $= x^{1/2}$

(a). Find the linearization $L(x)$ at $x = 64$.

① pt $f(64) = \sqrt{64} = 8$

② slope $f'(x) = \frac{1}{2}x^{-1/2} = \frac{1}{2\sqrt{x}}$

$f'(64) = \frac{1}{2\sqrt{64}} = \frac{1}{2 \cdot 8} = \frac{1}{16}$

$L(x) = f(64) + f'(64)(x-64)$

$\boxed{L(x) = 8 + \frac{1}{16}(x-64)}$

(b). Use the linearization from part (a) to approximate $\sqrt{64.2}$. i.e. Use $L(x)$ to approximate $f(64.2)$.

[You do **not** need to simplify the approximation in part (b).... Seriously, don't simplify it.]

$\sqrt{64.2} = f(64.2) \approx L(64.2) = \boxed{8 + \frac{1}{16}(64.2 - 64)}$

5. (~~10~~ 8 pts). Newtons Law of Gravitation says that the magnitude F of the force exerted by a body of mass m on a body of mass M is

$F = \dfrac{GmM}{r^2}$ $= GmMr^{-2}$ where G is the gravitational constant and r is the distance between the bodies.

(a). Find and simplify $\dfrac{dF}{dr}$.

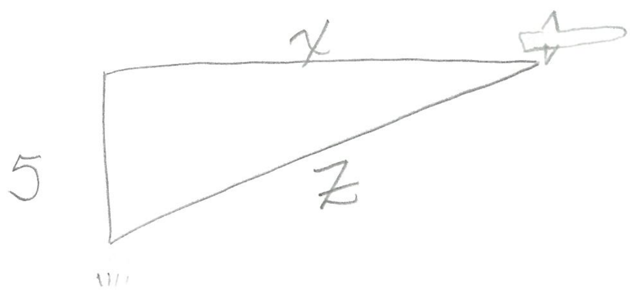
$$\boxed{\frac{dF}{dr} = -2GmMr^{-3} = -\frac{2GmM}{r^3}}$$

(b). Explain (briefly) the meaning of $\dfrac{dF}{dr}$.

$\frac{dF}{dr}$ is the rate of change of the force exerted as the radius between the objects changes

6. (14 pts). A plane flying horizontally at an altitude of 5 mi and a speed of 480 mi/h passes directly over a radar station. Find the rate at which the distance from the plane to the station is increasing when the distance from the plane to the station is 6 mi.

[Remember that significant partial credit will be given for clearly and accurately labeling the picture, and indicating values and equations in correct mathematical notation.]



x = distance plane flies

$\frac{dx}{dt}$ = velocity of plane

z = distance between plane & radar station

$\frac{dz}{dt}$ = rate the distance between changes.

Find $\frac{dz}{dt}$ when $z = 6$

$\frac{dx}{dt} = 480$

$$x^2 + 5^2 = z^2$$

$$\frac{d}{dt}[x^2 + 25] = \frac{d}{dt}[z^2]$$

$$\cancel{2}x\frac{dx}{dt} = \cancel{2}z\frac{dz}{dt}$$

$$x\frac{dx}{dt} = z\frac{dz}{dt}$$

$$\frac{dz}{dt} = \frac{x}{z}\frac{dx}{dt}$$

$$= \frac{\sqrt{11}}{\cancel{6}} \cdot \overset{80}{\cancel{480}}$$

$$= \boxed{80\sqrt{11}\ mi/h}$$

still need x at same instant:

$$x^2 + 5^2 = 6^2$$

$$x^2 = 36 - 25$$

$$x^2 = 11$$

$$x = \pm\sqrt{11} \Rightarrow x = \sqrt{11}$$

[Include units in your answer.]

7. (12 pts). Given $f(x) = (x^2 - 1)^3$ find the absolute maximum and absolute minimum values of f on the closed interval $[-1, 2]$.

$f'(x) = 3(x^2-1)^2 \cdot 2x = 6x(x^2-1)^2 = 0$

$6x = 0$ or $x^2 - 1 = 0$

$x = 0$ $\quad x = \pm 1$

At Crit #'s

$f(0) = (0^2-1)^3 = -1$

$f(1) = (1^2-1)^3 = 0$

$f(-1) = ((-1)^2-1)^3 = 0$

also endpt.

At Endpts

$f(2) = (2^2-1)^3 = 3^3 = 27$

Abs. max = 27 @ $x = 2$
Abs. min = -1 @ $x = 0$

8. (14 pts). Given $f(x) = 2x^4 - 3x^2 + 4$,

(a). Find all intervals on which f is concave up or down.

$f'(x) = 8x^3 - 6x$

$f''(x) = 24x^2 - 6 = 0$

$6(4x^2-1) = 0$

$4x^2 = 1$

$x^2 = \frac{1}{4}$

$x = \pm\frac{1}{2}$

$f'' = 6(4x^2-1)$

+	−	+
(+)(+)	(+)(−)	(+)(+)
$-\frac{1}{2}$		$\frac{1}{2}$
$x=-10$	$x=0$	$x=10$

Concave up: $(-\infty, -\frac{1}{2}) \cup (\frac{1}{2}, \infty)$

Concave down: $(-\frac{1}{2}, \frac{1}{2})$

(b). Find the location(s)(i.e. x-coordinate(s)) of all inflection points. [Do not find the y-coordinate(s).]

At $x = -\frac{1}{2}, \frac{1}{2}$