1. Solve the following inequalities. Write your solution in interval notation and sketch it on the numberline.

(a). $2x^2 + x \geq 3$ $(-\infty, -\frac{3}{2}] \bigcup [1, \infty)$

(b). $\dfrac{(x-3)^2(x+1)}{x+4} \geq 0$ $(-\infty, -4) \bigcup [-1, \infty)$

2. Section 1.1, #10 No, it is not a function. It fails the VLT.

3. Given $f(x) = \begin{cases} -1, & x \leq -1 \\ x, & -1 < x \leq 2 \\ x^2+1, & x > 2 \end{cases}$

(a). Find the domain and sketch the function. domain: All real numbers

(b). Find $f(-2)$, $f(2)$, and $f(4)$. $f(-2) = -1$, $f(2) = 2$, $f(4) = 17$

4. Find the domain of $f(x) = \dfrac{1}{\sqrt{x^2+x}}$. $(-\infty, -1) \bigcup (0, \infty)$

5. Section 1.4 #7

6. Section 1.5 #7, 9, 17

7. Evaluate the following limits, if they exist (clearly indicate $+\infty$ or $-\infty$ in the case of an infinite limit). If the limit does not exist, **explain the reason why**.

(a). $\lim_{x \to 0} \dfrac{x-3}{x(x+4)}$ DNE (one-sided limits are different)

(b). $\lim_{x \to -4} \dfrac{x^2+2x-8}{x+4} = -6$

(c). $\lim_{x \to 0} \dfrac{\sqrt{3x^2+4}}{x-4} = -\dfrac{1}{2}$

(d). $\lim_{x \to 0} \dfrac{x-3}{x^2(x+4)} = -\infty$

(e). $\lim_{h \to 0} \dfrac{\sqrt{x+h} - \sqrt{x}}{h} = \dfrac{1}{2\sqrt{x}}$

(f). $\lim_{x \to 1} f(x)$, where $f(x) = \begin{cases} 3, & x \leq 1 \\ 1, & x > 1 \end{cases}$
DNE (one-sided limits are different)

8. Given that $\lim_{x \to 1} f(x) = 2$, $\lim_{x \to 1} g(x) = 4$, $\lim_{x \to 1} h(x) = 0$, find the following limits if they exist.

(a). $\lim_{x \to 1} f(x) - g(x) = -2$

(b). $\lim_{x \to 1} f(x) \cdot g(x) = 8$

(c). $\lim_{x \to 1} h(x)/g(x) = 0$

(d). $\lim_{x \to 1} g(x)/h(x)$ DNE (One-sided limits are $+\infty$ or $-\infty$, but not enough info to determine which one or if they agree.)

9. For each of the following functions,
(i) find all of the x-values, if any, where g(x) is discontinuous and
(ii) indicate whether it is a removable, infinite, or jump discontinuity.

(a). $g(x) = \dfrac{x^2 - x - 6}{x(x^2 - 9)}$ infinite at $x = 0$, infinite $x = -3$, removable at $x = 3$

(b). $f(x) = \begin{cases} x^2 - 1, & x \leq 1 \\ 1 - x & x > 1 \end{cases}$ Continuous everywhere

10. Given the function $f(x) = x^3 - 2x^2 + 8x - 1$, use the Intermediate Value Theorem to show that there is a number c where $0 < c < 2$, such that $f(c) = 6$. $f(0) = -1$ and $f(2) = 15$. Since $-1 \leq 6 \leq 15$ AND f is continuous, then the IVT guarantees $f(x)$ must pass through $y = 6$ for some value of $x = c$ in the interval $(0, 2)$.

11. Suppose $f(1) = 3, f'(1) = -2, f(5) = 8$, and $f'(5) = 15$. Let P be the point on the graph $y = f(x)$ where $x = 1$. Let Q be the point on the graph of $y = f(x)$ where $x = 5$.

(a). Find the equation of the secant line PQ. $y - 3 = \frac{5}{4}(x - 1)$

(b). Find the equation of the tangent line to $y = f(x)$ at P. $y - 3 = -2(x - 1)$

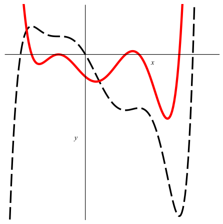
12. Section 2.1 #49 $T'(8)$ represents the *rate* at which the temperature is changing at 8am. $T'(8) \approx 3.75°$ F/hour.

13. Given $f(x) = x^2 - x$, ***use the limit definition*** $\displaystyle\lim_{x \to a} \frac{f(x) - f(a)}{x - a}$ to find the slope of the tangent line to the curve at $x = 2$. Use your result to write an equation of the tangent line at $x = 2$. $m = 3$, $y - 2 = 3(x - 2)$

14. Suppose the position of a particle at time t seconds is given by $s(t) = \sqrt{t}$ meters. ***Use the limit definition*** to find the velocity of the particle at time $t = 5$. $v(5) = \dfrac{1}{2\sqrt{5}}$

15. ***Use the limit definition*** of the derivative to find $f'(x)$ for $f(x) = \dfrac{1}{x^2}$. [You **must** use the limit definition.] $f'(x) = -\dfrac{2}{x^3}$

16. Both the function f and its derivative f' are plotted on the same set of axes below. Which curve represents the function and which curve represents the derivative? ***Justify your answer.*** dashed (black) is $f(x)$ solid (red) is $f'(x)$ Everywhere the dashed (black) curve has a horizontal tangent line (slope = 0), the solid (red) curve goes through 0.



17. Section 2.2 #3, 15, 39.

18. ***Use the limit definition*** $f'(a) = \displaystyle\lim_{h \to 0} \frac{f(a + h) - f(a)}{h}$ to show that $f(x) = |x - 5|$ is not differentiable at $x = 5$.
Simplify the limit and use the definition of $|x|$ to show that the one-sided limits are different.

19. Differentiate the following using ***Differentiation Rules***.

(a). $y = 10x^3 - 3x + 7$ $\quad y' = 30x^2 - 3$

(b). $y = (3x)^3$ $\quad \frac{dy}{dx} = 81x^2$

(c). $y = \dfrac{x + 4x^3 - 3}{x^4 + 3}$ $\quad y' = \dfrac{(x^4+3)\left(1+12x^2\right) - \left(x+4x^3-3\right)\left(4x^3\right)}{(x^4+3)^2}$

(d). $s(t) = t^2(3t - 4t^3)$ $\quad s'(t) = 9t^2 - 20t^4$

(e). $g(\theta) = \dfrac{\theta^3 \sin\theta}{2\theta + \sec\theta}$ $\quad \dfrac{(2\theta + \sec\theta)(\theta^3\cos\theta + (\sin\theta)3\theta^2) - \theta^3\sin\theta(2 + \sec\theta\tan\theta)}{(2+\sec\theta)^2}$

20. Find an equation of the tangent line to the curve $f(x) = \dfrac{3}{x^2} - \sqrt{x}$ at $x = 1$. $\quad y - 2 = -\frac{13}{2}(x-1)$

21. Find an equation of the tangent line to the curve $y = 3\tan x$ at the point $x = \dfrac{\pi}{3}$ $\quad y - 3\sqrt{3} = 4\left(x - \frac{\pi}{3}\right)$

22. Given the following information, find the values of the remaining trigonometric functions.

$\tan\theta = 3, \quad \pi < \theta < \dfrac{3\pi}{2}.$

$\sin\theta = -\frac{3}{\sqrt{10}}, \ \cos\theta = -\frac{1}{\sqrt{10}}, \ \tan\theta = 3, \ \csc\theta = -\frac{\sqrt{10}}{3}, \ \sec\theta = -\sqrt{10}, \ \cot\theta = \frac{1}{3}$

23. Given $\theta = \frac{3\pi}{4}$, find $\sin 2\theta$. $\quad -1$

24. Solve the following equations for x.

(a). $2\sin^2 x - \sqrt{2}\sin x = 0$ (x in $[0, 2\pi]$) $\quad x = 0, \pi, 2\pi, \frac{\pi}{4}, \frac{3\pi}{4}$

(b). $\cos\left(\dfrac{x}{2}\right) = 0$ (x in $[0, 2\pi]$) $\quad x = \pi$