Name: Key

Math 151, Calculus I – Crawford

Exam 1

24 September 2019

	Score
1	/10
2	/14
3	/8
4	/16
5	/14
6	/10
7	/10
8	/10
9	/4
10	/6
Total	/100

- Calculators, books, notes (in any form), cell phones, and any unauthorized sources are ***not*** allowed.
- You may use the given Unit Circle.
- Clearly indicate your answers.
- ***Show all your work*** – partial credit may be given for written work.
- ***Good luck***!

1. (10 pts). Find the domain of $f(x) = \sqrt{8x - 2x^2}$.

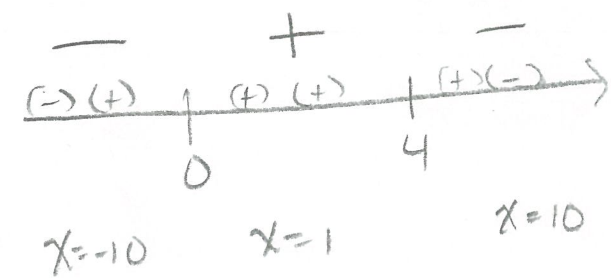
$8x - 2x^2 \geq 0$

$2x(4-x) \geq 0$ ie positive or 0.

Breaking Pts:

$2x = 0$ or $4 - x = 0$

$x = 0$ or $x = 4$



$\boxed{\text{Domain: } 0 \leq x \leq 4}$

or $[0,4]$

or

2. (14 pts). Evaluate the following limits, if they exist. Clearly indicate $+\infty$ or $-\infty$ in the case of an infinite limit. If the limit does not exist, **clearly explain the reason why**.

(a). $\lim_{x\to 9} \frac{\sqrt{x}-3}{x-9} \cdot \frac{\sqrt{x}+3}{\sqrt{x}+3} = \lim_{x\to 9} \frac{\cancel{x-9}}{\cancel{(x-9)}(\sqrt{x}+3)}$

$\frac{\sqrt{9}-3}{9-9}$

$\frac{0}{0}$

Ind. Form

$\Rightarrow$ More work

$= \lim_{x\to 9} \frac{1}{\sqrt{x}+3} = \frac{1}{\sqrt{9}+3}$

$= \frac{1}{3+3}$

$= \boxed{\frac{1}{6}}$

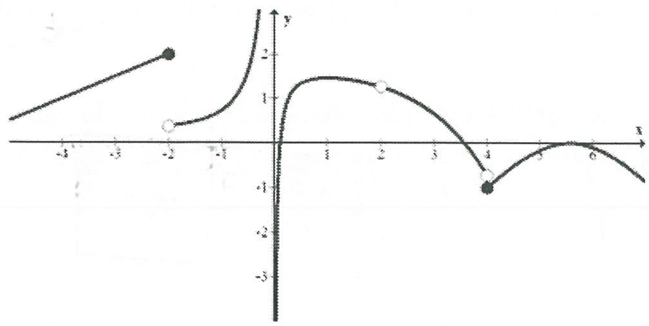
(b). $\lim_{x\to 2} \frac{x^2-2x}{x^2+3x-10} = \lim_{x\to 2} \frac{x\cancel{(x-2)}}{\cancel{(x-2)}(x+5)}$

$\frac{4-4}{4+6-10} \rightarrow \frac{0}{0}$

Ind. Form

$\Rightarrow$ More Work

$= \lim_{x\to 2} \frac{x}{x+5} = \boxed{\frac{2}{7}}$



3. (8 pts). Given the graph of $f(x)$ above.

(a). Is f continuous from the right at $x = -2$?

No

(b). State which type of discontinuity is at $x = 2$.

Removable

(c). Explain why the function is discontinuous at $x = 2$. i.e., Explain which of the three conditions from the definition of continuity do not hold. [Stating what type of discontinuity is not sufficient.]

Condition 1 fails since f(2) DNE.

[Therefore condition 3 $\lim_{x \to 2} f(x) = f(2)$ automatically fails since f(2) DNE]

(d). Is f differentiable at $x = 2$?

No

4. (16 pts). The position of a particle at time t seconds is given by $s(t) = \dfrac{12}{3+t}$ cm.

(a). Find the average velocity of the particle over the time interval $[1, 3]$. [Include units in your answer.]

$$v_{ave} = \frac{f(3)-f(1)}{3-1} = \frac{2-3}{3-1} = \boxed{\frac{-1}{2}}$$

$f(3) = \frac{12}{3+3} = 2$ pt $(3,2)$

$f(1) = \frac{12}{3+1} = 3$ pt $(1,3)$

(b). **Use the limit definition** $v(a) = \lim\limits_{t\to a} \dfrac{s(t)-s(a)}{t-a}$ or $v(a) = \lim\limits_{h\to 0} \dfrac{s(a+h)-s(a)}{h}$ to find the instantaneous velocity when $t = 1$. $a = 1$

[Include units in your answer.]

You must use the limit definition and you must show all of your work.

$$v(a) = \lim_{t\to 1} \frac{s(t)-s(1)}{t-1}$$

$$= \lim_{t\to 1} \frac{\left(\frac{12}{3+t} - \frac{12}{3+1}\right)}{t-1} = \lim_{t\to 1} \frac{\left(\frac{12}{3+t} - \frac{3}{1}\right)}{t-1}$$

$$= \lim_{t\to 1} \frac{\left(\frac{12-3(3+t)}{3+t}\right)}{\left(\frac{t-1}{1}\right)}$$

$$= \lim_{t\to 1} \frac{12-9-3t}{(3+t)} \cdot \frac{1}{t-1}$$

$$= \lim_{t\to 1} \frac{3-3t}{(3+t)(t-1)}$$

$$= \lim_{t\to 1} \frac{-3\cancel{(t-1)}}{(3+t)\cancel{(t-1)}} = \lim_{t\to 1} \frac{-3}{3+t}$$

[Note: $s'(t) = v(t) = -\frac{12}{(3+x)^2}$, if you want to check your answer.]

$$= \frac{-3}{3+1} = \boxed{-\frac{3}{4}}$$

OR

$$v(a) = \lim_{h\to 0} \frac{s(1+h)-s(1)}{h}$$

$$= \lim_{h\to 0} \frac{\left(\frac{12}{3+1+h} - \frac{12}{3+1}\right)}{h}$$

$$= \lim_{h\to 0} \frac{\left(\frac{12}{4+h} - \frac{12}{4}\right)}{h}$$

$$= \lim_{h\to 0} \frac{\left(\frac{12}{4+h} - 3\right)}{h}$$

$$= \lim_{h\to 0} \frac{\left(\frac{12-3(4+h)}{4+h}\right)}{\left(\frac{h}{1}\right)}$$

$$= \lim_{h\to 0} \frac{12-12-3h}{4+h} \cdot \frac{1}{h}$$

$$= \lim_{h\to 0} \frac{-3\cancel{h}}{(4+h)\cancel{h}} = \lim_{h\to 0} \frac{-3}{4+h}$$

$$= \boxed{-\frac{3}{4}}$$

For the remainder of the test, use the DIFFERENTIATION RULES to find any needed derivatives.

Do ***NOT*** use the limit definition.

5. (14 pts). Differentiate the following using Differentiation Rules. Do ***NOT*** use the limit definition!

[Do not simplify.]

(a). $y = 5x^4 + \frac{1}{3}x + x\sqrt{x}$ $= 5x^4 + \frac{1}{3}x + x^{3/2}$

$x\sqrt{x} = x^1 \cdot x^{1/2}$
$= x^{3/2}$

$$\boxed{y' = 20x^3 + \frac{1}{3} + \frac{3}{2}x^{\frac{1}{2}}}$$

(b). $f(x) = (x^2 + 4x - 3)\cos x$

$$\boxed{f'(x) = (x^2+4x-3)(-\sin x) + (\cos x)(2x+4)}$$

6. (10 pts). Find the first **and** second derivatives of $g(\theta) = \sec\theta$.

$$\boxed{g'(\theta) = \sec\theta \tan\theta}$$

$$\boxed{g''(\theta) = \sec\theta \cdot \sec^2\theta + \tan\theta \cdot \sec\theta \tan\theta}$$

$$= \sec^3\theta + \tan^2\theta \sec\theta$$

7. (10 pts). Find an equation of the tangent line to $y = \dfrac{x^2-1}{x+2}$ when $x = 2$.

① pt : $y = \dfrac{(2)^2-1}{2+2} = \dfrac{3}{4} \Rightarrow pt\left(2, \dfrac{3}{4}\right)$

② slope: $y' = \dfrac{(x+2)(2x)-(x^2-1)(1)}{(x+2)^2} = \dfrac{2x^2+4x-x^2+1}{(x+2)^2}$

$= \dfrac{x^2+4x+1}{(x+2)^2}$

$m = y'\Big|_{x=2} = \dfrac{2^2+4(2)+1}{(2+2)^2}$

$= \dfrac{13}{16} \Rightarrow \boxed{y - \dfrac{3}{4} = \dfrac{13}{16}(x-2)}$

8. (10 pts). Solve the following equation for all x.

$\cos^2(x) - \dfrac{1}{4} = 0$

$\cos^2 x = \dfrac{1}{4}$

$\cos x = \pm\sqrt{\dfrac{1}{4}}$

$= \pm\dfrac{1}{2}$

$x = \dfrac{\pi}{3} + 2\pi n$

or $x = \dfrac{2\pi}{3} + 2\pi n$

or $x = \dfrac{4\pi}{3} + 2\pi n$

$x = \dfrac{5\pi}{3} + 2\pi n$

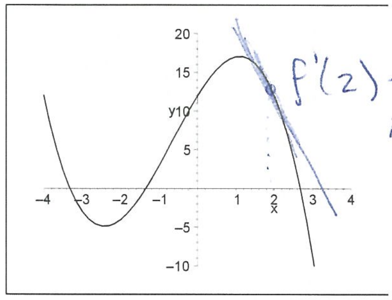
for $n = 0, \pm 1, \pm 2, \ldots$

9. (4 pts). *True or False.* Clearly indicate whether the following statements are true or false.

T F If $f(1) < 0$ and $f(4) > 0$, then there exists a number c in $(1, 4)$ such that $f(c) = 0$.

f must be continuous

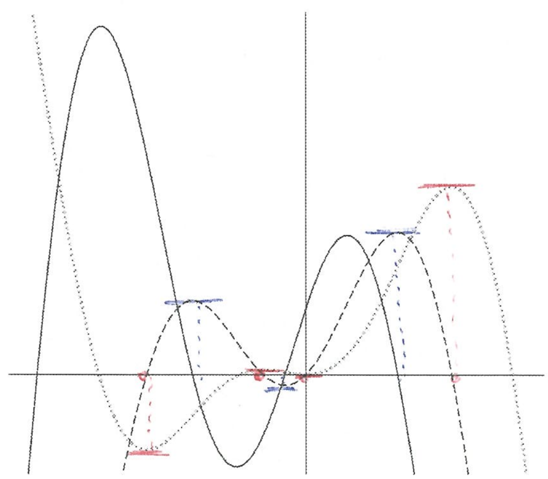
T F If the graph of a function $y = f(x)$ is given below, then the derivative $f'(2) > 0$.



$f'(2)$ = slope < 0

i.e neg. slope

10. (6 pts). The figure below shows the graph of f, f', and f''. Match the solid, dashed, and dotted curves to the correct function f, f', or f''.



[Fill in the blank with f, f', or f''.]

Solid: f''

Dashed: f'

Dotted: f

Dashed is Derivative of Dotted.

Solid is Derivative of Dashed